Prospective Teacher's Representations of Multiplication

Mohan Chinnappan
University of Wollongong
<mohan@uow.edu.au>

The use of ICT-based resources in order to provide effective learning environments in which children could explore concepts such as multiplication has received considerable attention. This approach is based on the assumption that teachers who are already teaching and those who are being trained to become teachers, draw on a well-developed knowledge of the multiplicative process, and could exploit the ICT appropriately. The aim of the study reported here was to examine the quality of content and pedagogical content knowledge of multiplication developed by a group of prospective elementary mathematics teachers in the context of an ICT-based software. Analysis of data showed the participants are developing solid understandings but some areas could be strengthened.

In recent years research in mathematics education has generated a substantial body of work on teacher education. Broadly, this stream of research has examined issues related to mathematics teacher development, change and professional practice. Investigations that focused on teacher development are beginning to focus on the nature of support that can be provided to teachers who are new to the profession. Teaching and learning to teach is a complex activity and entails varied experiences. The transition from being a novice to an experienced professional practitioner involves changes to the way they understand the subject and teaching of the subject.

The Issue and Significance

The measuring and modelling of mathematics teachers' practice and associated knowledge have been receiving increased attention from the research community. As teacher knowledge is not static, such research needs to consider the evolutionary nature of teacher knowledge and examine situations that promote the growth of rich content and pedagogical content knowledge as these relate to multiplication (Ball & Bass, 2000; Simon, 1995). Teachers develop alternative perspectives about elementary mathematics and this needs investigation. Indeed, the description of growth and change in teacher knowledge and the nature of experiences that contribute to this growth as been identified as a major challenge for researchers (Fennema & Franke, 1992; Shulman & Shulman, 2004). The research reported here describes the type of knowledge activated by participating prospective teachers at a particular point in this developmental path. The phenomenon under scrutiny is the type representations of multiplications that a group of future teachers construct with the aid of a Web-based cultural tool.

Conceptual Framework

The analysis of data in the present study was guided by activity theory. Prospective teachers' understandings of multiplication and the teaching of multiplication as they emerged through an activity are central to this view of learning. A key assumption here is that as participating prospective teachers engage in learning activities, the tools that are employed during these activities structures the nature of their participation and meanings constructed by the participants (Lave, 1988; Loent'ev, 1978). Any knowledge that an individual develops is seen as a product of and anchored by the tools that drive an activity. The notion of *tools* and *activity* are central to studies of classroom mathematics teaching and learning. According to activity theory, tools could act as affordances and constraints in teachers' attempts to develop meanings with mathematics concepts. The framework of activity permits researchers understand the dynamics that is involved in the construction of meanings as student teachers interact with a pedagogical tool.

Development of Multiplicative Structures

The understanding of multiplication and its applications are based on the quality of multiplicative structures or schemas that teachers construct. It would seem that the teaching of multiplication must assist children explore its different meanings and properties as a means to enriching the underlying multiplicative structures (Vergnaud, 1988). This outcome can be achieved by adopting a strategy in which teachers employ resources to model multiplication in different ways. The complex nature of multiplication is reflected in the number of models that one can construct.

Two models of multiplication are repeated addition and area/rectangular array. These macromodels are built on submodels which in turn are built on schemas of multiples and factors, grouping, properties of multiplication (commutative, associative, distributive) and multiplication algorithm. Repeated addition shows, for example, that 7 x 5 is equivalent to 7 + 7 + 7 + 7 + 7. It is important here for children to see the relationship between addition and multiplication. That is, multiplying 7 by 5 is the same thing as adding seven fives together. Modelling should aim to help children discover that adding seven fives together will give you the same result as adding five sevens (commutativity). The use of rectangular arrays provides an effective way to help children visualize multiplication, but this strategy should be grounded both in symbolic representation (7 + 7 + 7 + 7 + 7) and real-life contexts.

The foregoing analysis of multiplication raises an important issue about teacher knowledge that could influence not only modelling but also the appropriate use resources in order to model multiplication situations. Kaput (1992) argued that Information and Communication Technology (ICT)-related tools provided a dynamic learning environment to model and extend concepts and skills in the mathematics classroom. While this view has received considerable support from the teaching community, less is known about the nature of knowledge that teachers are able to activate in this medium.

Burgess and Bicknell (2003) expressed the view that research needed to investigate both teachers' subject-matter and pedagogical content knowledge that could drive their actions. This issue also featured prominently in arguments advanced by Brown and Borko (1992) that there is a need to examine the development of knowledge base of prospective teachers of mathematics at different phases of their professional education. I address this issue in the present study in two ways. Firstly, the study aims to identify the quality of pre-service teachers' subject-matter knowledge of multiplication, an area in K-6 mathematics that had proved to be difficult for this group (Clarkson, 1998; Tirosh & Graeber, 1989). Secondly, the present study attempts to generate data that is relevant to the debate about the relationship between the quality of teachers' subject-matter knowledge and the use of that knowledge to model multiplication within an ICT environment. Teachers who have built up a richer store of subject-matter (multiplication) and pedagogical content knowledge (modelling of multiplication) can be expected to exploit ICT more effectively than those with a weaker knowledge base. It was anticipated that the support provided by the ICT program would also prompt student teachers activate wider representations of multiplication operations.

The growth of understanding of multiplication could be characterized as involving a progressive change in the mixture of personal and formal knowledge. The models that teachers construct must aim to assist children bridge these two understandings. The most detailed analysis of multiplication was undertaken by Greer (1992). His analysis showed that multiplication knowledge consists of many interwoven strands. He identified a range of situations that involve multiplication. An important outcome of his framework is the specification of cognitive structures (subconstructs) that provide support for the maturing of multiplicative structures along different dimensions. While there is little doubt that the acquisition of the above subconstructs is important, children must also show evidence of understanding the relationships that exist among these clusters of knowledge. In an examination of the role of multiplication tables and concept development, Martin (2001) argued that a rich representation of multiplication knowledge should draw out key within- and between-concept relations and operations. This analysis placed emphasis on connections among the multiple representations of multiplication. There is, therefore, a need to generate information about the structure and organization of the bits of knowledge involved in multiplication.

$B_{10}B$ (Base 10 Blocks) Program

The researcher selected a learning tool that would allow teachers construct multiplicative models and test conjectures. This class of mathematical activity, according to Kaput (1992) allows the observation of translations between representations. The $B_{10}B$ program developed by Bulaevsky (1999) was chosen for the afore-mentioned purpose. The program consists of a panel as shown in Figure 1. On the left-hand side of the panel there are three different blocks each representing a unit, 10 units (*long*) and hundred units (*flat*) that can be dragged into the working panel. Teachers can then move, rotate, break, and glue the blocks to explore their relations. Clicking on the base-ten chart on the left allows prospective teachers to switch the backdrop in the working area. There are four backdrop areas and the XY backdrop shown in Figure 1 allows for work on multiplication. Teachers could use the *[100, 10, 1]* backdrop for work on place value ideas.

On the top row, there are eight icons. Icons 1-6 are useful for the performance of arithmetic operations. The hammer allows prospective teachers to break a *long* into *units.* The lasso helps teachers to group and move pieces within any part of the panel. The second icon on the row permits the rotation of any of the three blocks. The glue helps teachers group and create a shape with smaller blocks. In this manipulative system teachers could break apart the virtual blocks to decompose them into smaller blocks or glue groups of smaller blocks to make larger blocks. $B_{10}B$ encourages flexibility in prospective teachers' approach to creating and working with numbers. For example, if a child wants to make 89, she can pull out 8 *longs* (80) and 9 *units* (9) or she can pull out a *flat* (100), break it up so that she can use 90, and next break a *long* (from the 90) so that she can just use 9 *units*. These actions are based on understandings of groupings and regroupings that are consistent with the base-10 numeration system. The system can be utilised for the teaching of a variety of number-related concepts including division and multiplication. From the activity theory perspective, the tool affords the construction of pictorial representations of multiplication but places constraints on situating the concept in real-life contexts.

Method

Participants

The participants in the present study were a cohort of 15 prospective teachers. These student teachers were enrolled in the third year of their BEd (Primary) program. The student teachers have completed two mathematics methods subjects emphasizing constructivist principles in primary and early childhood mathematics teaching and learning. They were involved in six weeks of teaching practice before the commencement of the study. During the two years prior to the study, the participants satisfied the mathematics discipline requirements for the BEd (Primary), which included number, geometry and algebra. All of them had made use of computers during their courses, and exhibited reasonable levels of facility with ICT.

Tasks and Procedure

The investigator met the group on two occasions. During the first meeting, which lasted about sixty minutes, the participants were informed about the project and asked to revise previous work that examined teaching arithmetic skills to K-6 children. At this meeting, participants were encouraged to engage in a discussion about concepts that are relevant to teaching numbers and operations involving whole numbers. A number of previous tutorial activities in which the student teachers had explored the teaching of numbers were also revisited, including group discussions about the appropriate use of concrete material to help young children grasp numbers and operations. During this session, the participants were not asked to focus on multiplication.

During the second session, the participating student teachers were required to work in groups of two. Each group downloaded $B_{10}B$ from the Internet and explored its use as a teaching and learning tool in the classroom. Participants were given about 60 minutes to explore the $B_{10}B$ panel, and encouraged to raise questions. When the student teachers had indicated that they were happy and felt comfortable with $B_{10}B$ the investigator asked them to respond to two focus questions. Firstly, they were required to discuss and demonstrate the use of the program teach multiplication involving 1-digit and 2-digit numbers to third and fourth graders. The second question also asked the participants about how the program could be utilized to illustrate multiplication in real-life contexts. Participants' were asked to record their responses on the paper sheets that were provided and save relevant computer files. This second part of session 2 lasted between 60-80 minutes.

The transcripts were then analyzed for evidence of two groups of knowledge: content knowledge about multiplication (Table 1) and teaching of this concept (Table 2). The former also included student teachers' articulation of properties of multiplication. The latter knowledge component examined the representational features of multiplication that would help children grasp and develop further insights. Taken together, these two components provided insight into the subject matter and pedagogical content knowledge of student teachers that would significantly influence the type of scaffolds that the teachers could provide for children in the classroom. One feature of this analysis was the links that participating student teachers made among the above components of their knowledge base. The links were considered to be an additional index of richness of participants' pedagogical content knowledge. Evidence of prospective teachers' knowledge was scored as follows: 0

– not activated, 1 – incorrect use or interpretation of the concept, 2 – partly correct use or interpretation of the concept and 3 – correct use or interpretation of the concept.

Results

The means and standard deviation scores for participants' content knowledge of multiplication are presented in Table 1. Respondents have a well-developed understanding of place value and repeated addition properties of multiplication of whole numbers. The prospective teachers were less forthcoming in articulating the distributive and rectangular/area nature of the focus concept. This pattern was also evident in the number of connections that were constructed. None of the participants showed an understanding of the commutative property of multiplication.

Table 1
Scores for Content Subconstructs

	Mean	Standard Deviation
Place value	2.87	0.52
Distributive property	0.27	0.80
Commutative property	0.00	0.00
Repeated addition	2.67	0.72
Rectangular array	0.67	0.90
Other connections	0.80	0.77

Further analysis of data on multiplication concepts and their modelling is presented in Table 2. The results here focused on the pedagogical aspects of the subconstructs that were identified in Table 1. That is, the scoring in Table 2 reflected the prospective teachers' ability to use the environment to model the above subconstructs. As expected, there was high degree of success in modelling the place value of numbers that were involved in the multiplicative process. A similar pattern was also noted in the teachers' ability to visualize the distributive character of the operation. All the remaining subcontructs were not represented in pedagogically clear or meaningful ways.

Table 2
Scores for Elaboration of Content Subconstructs

	Mean	Standard Deviation
Place value	2.87	0.52
Distributive property	0.27	0.80
Commutative property	0.00	0.00
Repeated addition	2.47	0.74
Rectangular array	0.47	0.52
Other connections	0.67	0.72

Figure 1 shows the actions of one student teacher (Emma) in her attempt to model the distributive property of multiplication. In this episode, she attempted to show the following relation: 12 x 5 = (10 +2) x 5 = (10 x 5) + (2 x 5) = 50 + 10 = 60. This student teacher made effective use of not only the blocks but also the base-10 chart on the left, which generated the *x*- and *y*- axis on the panel. She showed 12 as 10 plus 2 on the y-axis by using one *long* and two units. Likewise, she placed 5 units on the *x*-axis. She also explained that the product could be grouped by using the lasso into 5 *longs* and 2 sets of 5 units (one *long*), forming 60 (6 *longs*). This student teacher also commented that it would be useful to be able to write the numbers along side the blocks thus indicating a desire to integrate symbolic representation within her model.

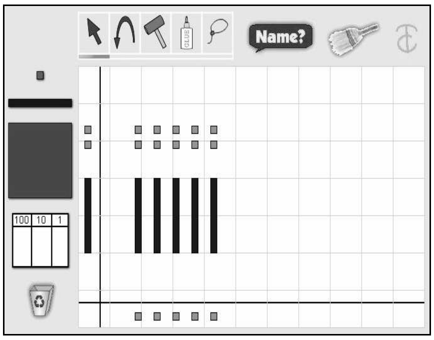

Figure 1: Elaboration of the distributive property by Emma.

Discussion and Implications

This study explored two components of the knowledge base of prospective teachers (understanding of multiplication and the teaching of the concept) with the aid of an ICT-supported tool. It was hypothesized that student teachers with a deeper insight into multiplication as a concept (as reflected in the multiple representations) and situations involving multiplications would make more effective use of the ICT resource.

The participants could draw on the place-value backdrop (a feature of the tool) and make use of the *units* and *longs* to create numbers that were involved in the multiplicative process. None of the participants used the *flat* to demonstrate the multiplication of single and double-digit numbers. They could have used the *flats* to articulate general properties of multiplication but the results did not bear this. The student teachers were more focused on showing that multiplication of two whole numbers could produce a third number that was larger than the initial two numbers. The increase in the size of the product was also portrayed in the models that were constructed by the participants. Analysis of repeated addition and its modelling indicated that this was a dominant representation. This constitutes an important understanding as it relates to the algebra underlying multiplicative operations. By grouping and gluing the blocks, and placing them on the place-value chart, the teachers also showed competency in demonstrating increase in the size of the product. The above approach could provide children with an opportunity to 'see' the connection between numeration and the computational process that was considered to be pivotal in understanding numbers and operations (Hiebert & Wearne, 1992, Mousley, 2000).

The ability to visualize multiplication as repeated addition in the $B_{10}B$ environment addresses two key learning issues raised by Schwartz (1988) in relation to difficulties that could be experienced by young children when they shift from dealings with addition to multiplication. Unlike addition and subtraction, in multiplication situations children are expected to work with composite units as opposed to single units. Additionally, multiplication may involve either like or unlike quantities to produce a third quantity (the product). This awareness of the relations between multiplier and multiplicand could facilitate children make the transition from their earlier experiences with addition and subtraction to multiplication.

A number of students' comments revolved around the construction of rectangular arrays, which provided windows into their thinking about how this artefact can be used to reinforce children's ability to visualise multiplication. The prospective teachers were also inclined to help children by drawing on ideas generated by other children in their classroom. This aspect of their pedagogical knowledge allowed for the crossing of the boundaries of meaning that individual children could construct with arrays and other models of multiplication.

While there was some evidence of generating an array model of multiplication, a significant proportion of the student teachers did not exploit $B_{10}B$ for this purpose. This could be due to a lack of knowledge about this form of representation of multiplication. The rectangular array model could also be used effectively to demonstrate commutative properties.

The participants were invited to relate the numbers that were involved in the operation with real-life situations. For example, problems such as 'Five children have 10 balloons each. How many do they have altogether?' can be solved by showing the one-to-one relationship between balloons and children with blocks in $B_{10}B$. However, none of the participants linked $B_{10}B$ with real-life multiplication problems. Perhaps, this could be due to participants becoming too involved with the manipulative aspects of the tool in question. Situating the concept is a key learning goal for classroom practices that aim for deep understanding of multiplication (Izsak, 2004). Knowledge about situating multiplicative concepts in contexts that are meaningful for children is necessary in order for teachers to display the type of multiplicative structures such as *isomorphism of measures* and *product of measures* that were identified by Vergnaud (1988). According to activity theory, tools could support or constrain teachers' knowledge constructions. In the present study, it appears that $B_{10}B$ had distracted students somewhat to focus on the more 'mathematical' aspects of multiplication. The data reported here does not allow one to make judgements about the context-based knowledge of multiplication developed by the participants. Future studies need to examine this issue of what teachers consider to be important in the context of using such tools more explicitly.

The cohort of student teachers in the present study had constructed a level of understanding that might be regarded as sufficient to introduce children to the procedural aspects of multiplication but may not be adequate to immerse children more deeply in concepts such as the inverse relations of multiplication to division. However, the trajectories of their understanding ought to be followed by conducting similar studies at different points in their professional development.

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